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Intermediate-Range Forces?

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It is shown that an intermediate-range variation in the gravitational "force" can be obtained through a generalized scalar-tensor theory of gravity.

The possibility of intermediate-range interactions (meters to kilometers) has been relatively unexplored (Fitch *et al.*, 1988). Experimental evidence and theoretical arguments suggest that such interactions may exist. Deep mine measurements of the gravitational force at various depths have given values of the gravitational constant G which may not agree with determinations at the surface (Long, 1976; Scherk, 1979, 1980; Gibbons and Whiting, 1981; Stacy and Tuck, 1981; Holding and Tuck, 1984; Holding *et al.*, 1986). The recent reanalysis of the Eötvös experiment (Eötvös *et al.*, 1922) by Fischbach *et al.* (1986) has given evidence for intermediate-range forces which gives rise to a reduction in the apparent gravitational constant over intermediate distances. To account for this possible difference, they postulate the existence of a new force.

Should this intermediate-range variation in the gravitational constant be confirmed, it can be explained via a purely gravitational (scalar tensor) theory (Cohen, 1964). This theory is based on the Lagrangian

$$L = A_1 R + A_2 \phi^{,\alpha} \phi_{,\alpha} + A_3 + A_4 L_M + A_5 \phi_{,\alpha}^{;\alpha} \tag{1}$$

where R is the scalar curvature, $\phi_{,\alpha}$ is the derivative of a scalar field ϕ with respect to x^α, the A's are arbitrary functions of ϕ, and L_M is the Lagrangian of matter, electromagnetic fields, etc. Any Lagrangian L of the form given by equation (1) has the property that under conformal transformations (Synge, 1960) (i.e., rescaling) of the metric $g_{\mu\nu}$, L is mapped onto another Lagrangian of the same general form. In Einstein's theory of gravity (Einstein, 1916, 1955) A_3 gives rise to the cosmological constant. Here A_3 is a function of ϕ, which can also give rise to a mass for the scalar field.

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The field equations of the theory are obtained by varying the action integral

$$I = \int d^4x\sqrt{-g}\, L \tag{2}$$

with respect to ϕ and $g^{\mu\nu}$, respectively.

One notes that

$$A_5\phi_{,\alpha}^{;\alpha} = (A_5\phi_{,\alpha})^{;\alpha} - A_5'\phi^{,\alpha}\phi_{,\alpha} \tag{3}$$

where the prime denotes differentiation with respect to ϕ. For a large class of A_5, the first term on the rhs of equation (3) can be converted to a vanishing surface integral at infinity, when substituted into equation (2). At this point restrict attention to those Lagrangians L^* for which A_5 has this property. Since the A's are arbitrary, a Lagrangian of the form

$$L = A_1R + A_2\phi^{,\alpha}\phi_{,\alpha} + A_3 + A_4L_M \tag{4}$$

gives rise to the same action as L^*.

Varying the action integral I for L given by equation (4) with respect to ϕ gives

$$2A_2\phi_{;\alpha}^{;\alpha} = A_1'R - A_2'\phi^{,\alpha}\phi_{,\alpha} + A_3' + A_4'L_M \tag{5}$$

Varying the action integral I with respect to $g^{\mu\nu}$ gives

$$0 = A_1G_{\mu\nu} + A_2(\phi_{,\mu}\phi_{,\nu} - \tfrac{1}{2}g_{\mu\nu}\phi^{,\alpha}\phi_{,\alpha}) - \tfrac{1}{2}A_3g_{\mu\nu} - \tfrac{1}{2}A_4T_{\mu\nu} - (A_{1;\mu\nu} - g_{\mu\nu}A_{1;\alpha}^{;\alpha}) \tag{6}$$

where, for those functions A_1 which do not change sign, without loss of generality, we can rescale $g_{\mu\nu}$ such that

$$A_1 = K + \phi \tag{7}$$

with K a constant. Below, this expression for A_1 is employed.

Contraction gives

$$-A_1R - A_2\phi^{,\alpha}\phi_{;\alpha} - 2A_3 + 3\phi_{;\alpha}^{;\alpha} - \tfrac{1}{2}A_4T = 0 \tag{8}$$

Eliminating R by substituting equation (8) into equation (5) yields

$$\left(2A_2 - \frac{3A_1'}{A_1}\right)\phi_{;\alpha}^{;\alpha} = \phi^{,\alpha}\phi_{,\alpha}\left(-\frac{A_1'}{A_1}A_2 - A_2'\right) + \left(A_3' - 2A_3\frac{A_1'}{A_1}\right) + A_4'L_M - \frac{A_1'A_4}{2A_1}T \tag{9}$$

If we now choose A_2 such that

$$\frac{A_1'}{A_1} + \frac{A_2'}{A_2} = 0 \tag{10}$$

this implies

$$A_2A_1=\bar{K} \tag{11}$$

where $\bar{K}$ is a constant. Hence the $\phi^{,\alpha}\phi_{,\alpha}$ term in equation (9) vanishes. Substituting equation (11) into the first parenthesis in equation (9) yields

$$2A_2-\frac{3A_1'}{A_1}=\frac{2\bar{K}-3}{A_1} \tag{12}$$

When this is substituted into equation (9), we obtain

$$(2\bar{K}-3)\phi_{;\alpha}^{;\alpha}-(A_1A_3'-2A_1'A_3)=-\tfrac{1}{2}A_4T+A_1A_4'L_M \tag{13}$$

A number of special cases may be of interest. If $A_3=\beta\phi^2+\gamma\phi+K\gamma/2$, with β and γ arbitrary constants, the third parenthesis in equation (9) gives

$$A_1A_3'-2A_1'A_3=(2K\beta-\gamma)\phi \tag{14}$$

Also, if we set

$$A_4'=0 \tag{14a}$$

that implies (Eddington, 1960)

$$T^{\mu\nu}_{;\nu}=0$$

If both of those assumptions are made, we can obtain a Klein-Gordon equation for ϕ

$$\phi_{;\alpha}^{;\alpha}-m^2\phi=-BT \tag{15}$$

if $m^2>0$ and

$$m^2=\frac{2K\beta-\gamma}{2\bar{K}-3}$$

$$B=\frac{A_4/2}{2\bar{K}-3} \tag{16}$$

For weak fields, and in the free space exterior to a spherical body, equation (15) can give a Yukawa-like falloff in ϕ. Here m may be determined experimentally. If m is of intermediate range, e.g., between 10^{-1} and $10^{-7}\,\text{m}^{-1}$, then light-bending, perihelion advance, and other planetary experiments will give results in agreement with general relativity.

Equation (6) gives the other set of field equations

$$G_{\mu\nu}=\frac{1}{2}\frac{A_4}{A_1}T_{\mu\nu}-\frac{A_2}{A_1}\left(\phi_{,\mu}\phi_{,\nu}-\frac{1}{2}g_{\mu\nu}\phi^{,\alpha}\phi_{,\alpha}\right)$$
$$+\frac{1}{2}\frac{A_3}{A_1}g_{\mu\nu}+\frac{1}{A_1}(A_{1;\mu\nu}-g_{\mu\nu}A_{1;\alpha}^{;\alpha}) \tag{17}$$

Equations (17) and (13) constitute the general equations of the theory. If $A_3=0$, $K=0$, and A_4 is constant, we recover the Brans-Dicke scalar tensor theory (Brans and Dicke, 1961; Jordan, 1955, 1959). If ϕ is constant and $A_3 = 0$, we recover Einstein's original general relativity equations (Gibbons and Whiting, 1981). If ϕ is constant and $A_3 \neq 0$, we recover Einstein's equations with nonvanishing cosmological constant. Other related work (Minkowsky, 1977; Zee, 1981; Bekenstein, 1986) involving Newton's gravitational constant, the Klein-Gordon equation, and a variational principle may also be of interest, since Newton's constant and the masses of all fields are generated spontaneously via a symmetry-breaking mechanism in a quantized theory. For an excellent review of massive scalar-tensor theories see Wagoner (1970) (see also Acharya and Hogan, 1973).

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